Inflation at Low Scales: General Analysis and a Detailed Model

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Abstract

Models of inflationary cosmology based on spontaneous symmetry breaking typically suffer from the shortcoming that the symmetry breaking scale is driven to nearly the Planck scale by observational constraints. In this paper we investigate inflationary potentials in a general context, and show that this difficulty is characteristic only of potentials $V(\phi)$ dominated near their maxima by terms of order ϕ^2. We find that potentials dominated by terms of order ϕ^m with $m > 2$ can satisfy observational constraints at an arbitrary symmetry breaking scale. Of particular interest, the spectral index of density fluctuations is shown to depend only on the order of the lowest non-vanishing derivative of $V(\phi)$ near the maximum. This result is illustrated in the context of a specific model, with a broken SO(3) symmetry, in which the potential is generated by gauge boson loops.

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I. INTRODUCTION

Inflation is an attractive model of the early universe because it naturally explains the smoothness and flatness of the universe, and provides a well-defined mechanism for the creation of primordial density fluctuations. In a typical inflationary model, spontaneous symmetry breaking at some scale v creates a potential for a scalar field with nonzero vacuum energy density Λ^4, resulting in an epoch in which the universe is dominated by vacuum energy and undergoes a period of exponential expansion. Quantum fluctuations in the scalar field create density fluctuations which later collapse to form galaxies and clusters of galaxies [1–4]. Although the inflationary paradigm is widely accepted, most attempts to incorporate inflation into specific models of particle physics suffer from two shortcomings: (1) Parameters such as coupling constants must often be "fine-tuned" to extremely small values in order to avoid massive overproduction of density fluctuations. This can be seen in a general sense to arise from an exponential dependence of the vacuum energy density Λ^4 on the symmetry breaking scale v. (2) Symmetry breaking scales significantly below the Planck scale, $v < m_{Pl} \simeq 10^{19} \text{GeV}$ are inconsistent with observational constraints. The latter difficulty is especially troubling, since physics at the Planck scale is currently poorly understood, and there is no compelling reason to expect that standard notions of spontaneous symmetry breaking are valid at such high energies.

Section II of this paper provides a brief review of slow-roll and inflationary constraints. In section III we show, using a general class of scalar field potentials, that the problems of fine-tuning and of scales being driven close to m_{Pl} are in fact characteristic only of a restricted class of potentials, those which are dominated by terms of order ϕ^2 in the scalar field driving inflation. We obtain the result that for potentials dominated by terms of order ϕ^m with $m > 2$, inflation can take place consistent with cosmological constraints at an arbitrary symmetry breaking scale. Of particular interest, we show that for $m > 2$, the spectral index n_s of the scalar density fluctuations is *independent* of the specific form of the potential, and is determined entirely by the order m of the lowest non-vanishing derivative

at the maximum of the potential, with $0.93 < n_s < 0.97$ for all orders m. The results of this section have been briefly reported earlier [5]. In section IV, we construct a specific model in which the inflationary potential is created by radiative corrections from gauge bosons in a Lagrangian with an explicitly broken SO(3) symmetry. This model is dominated by terms of order ϕ^4, and demonstrates the general result in a detailed context.

II. SLOW-ROLL AND INFLATIONARY CONSTRAINTS

A. Inflationary dynamics and constraints

Inflationary cosmologies explain the observed flatness and homogeneity of the universe by postulating the existence of an epoch during which the energy density of the universe was dominated by vacuum energy, resulting in a period of exponential increase in the scale factor of the universe [6–8],

$$a(t) \propto e^{Ht}. \tag{2.1}$$

The *Hubble parameter* H is given by

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3m_{Pl}^2}\rho_{vac} \simeq const., \tag{2.2}$$

where $m_{Pl} \simeq 10^{19}$GeV is the Planck mass. Nonzero vacuum energy is introduced into particle physics models by including a scalar field ϕ, the *inflaton*, with a potential $V(\phi)$. During inflation, the inflaton is displaced from the minimum of its potential, resulting in a nonzero vacuum energy, and evolves to the minimum with equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \tag{2.3}$$

So-called "new" inflationary models [7,8] consider potentials which contain at least one region flat enough that the evolution of the field is friction dominated, and the equation of motion can be taken to be:

$$3H\dot{\phi} + V'(\phi) = 0. \tag{2.4}$$

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This is known as the *slow-roll* approximation. This approximation can be shown to be valid if the *slow-roll parameters* ϵ and $|\eta|$ [9] are both less than 1, where:

$$\epsilon\left(\phi\right) \equiv \frac{m_{Pl}^2}{16\pi}\left(\frac{V'\left(\phi\right)}{V\left(\phi\right)}\right)^2,\tag{2.5}$$

and

$$\eta\left(\phi\right) \equiv \frac{m_{Pl}^2}{8\pi}\left[\frac{V''\left(\phi\right)}{V\left(\phi\right)} - \frac{1}{2}\left(\frac{V'\left(\phi\right)}{V\left(\phi\right)}\right)^2\right].\tag{2.6}$$

Consider a scalar field at an initial value ϕ. The field evolves according to equation (2.4) to the minimum of the potential, where it oscillates and decays into other particles (*reheating*). Inflation ends and reheating commences at a field value ϕ_f where the first order parameter $\epsilon\left(\phi_f\right)$ is unity [9]:

$$\epsilon\left(\phi_f\right) \equiv \frac{m_{Pl}^2}{16\pi}\left(\frac{V'\left(\phi_f\right)}{V\left(\phi_f\right)}\right)^2 = 1,\tag{2.7}$$

where $\epsilon\left(\phi\right) < 1$ during the inflationary period. The number of e-folds of inflation which occur when the field evolves from ϕ to ϕ_f is

$$N\left(\phi\right) = \frac{8\pi}{m_{Pl}^2}\int_{\phi_f}^{\phi}\frac{V\left(\phi'\right)}{V'\left(\phi'\right)}d\phi'.\tag{2.8}$$

Smoothness on scales comparable to the current horizon size requires $N \geq 60$, which places an upper limit on the initial field value $\phi \leq \phi_{60}$, where $N\left(\phi_{60}\right) \equiv 60$. Quantum fluctuations in the inflaton field produce density fluctuations on scales of current astrophysical interest when $\phi \simeq \phi_{60}$. The scalar density fluctuation amplitude produced during inflation is given by [10]:

$$\begin{aligned}\delta &= \left(\frac{2}{\pi}\right)^{1/2}\frac{[V\left(\phi_{60}\right)]^{3/2}}{m_{Pl}^3 V'\left(\phi_{60}\right)}\left\{1 - \epsilon\left(\phi_{60}\right) + \left(2 - \ln 2 - \gamma\right)\left[2\epsilon\left(\phi_{60}\right) - \eta\left(\phi_{60}\right)\right]\right\}\\ &\simeq \left(\frac{2}{\pi}\right)^{1/2}\frac{[V\left(\phi_{60}\right)]^{3/2}}{m_{Pl}^3 V'\left(\phi_{60}\right)},\end{aligned}\tag{2.9}$$

where $\gamma \simeq 0.577$ is Euler's constant. The amplitude of tensor, or gravitational wave, fluctuations is [10],

$$a_T = \frac{2}{\sqrt{3\pi}}\frac{[V\left(\phi_{60}\right)]^{1/2}}{m_{Pl}^2}\left[1 + \left(1 - \ln 2 - \gamma\right)\epsilon\left(\phi_{60}\right)\right]$$

$$\simeq \frac{2}{\sqrt{3\pi}} \frac{[V(\phi_{60})]^{1/2}}{m_{Pl}^2}. \tag{2.10}$$

In addition, it is possible to calculate the spectral index n_s of the scalar density fluctuations. The fluctuation power per logarithmic interval $P(k)$ is defined in terms of the density fluctuation amplitude δ_k on a scale k as $P(k) \equiv |\delta_k|^2$. The *spectral index n_s* is defined by assuming a simple power-law dependence of $P(k)$ on k, $P(k) \propto k^{n_s}$. The spectral index of density fluctuations, n_s, is given in terms of the slow-roll parameters ϵ and η [10]:

$$n_s \simeq 1 - 4\epsilon(\phi_{60}) + 2\eta(\phi_{60}). \tag{2.11}$$

For $\epsilon, |\eta| \ll 1$ during inflation, inflationary theories predict a nearly scale-invariant power spectrum, $n_s \simeq 1$.

In order for an inflationary model to be viable, it must satisfy observational constraints from the Cosmic Background Explorer (COBE) Differential Microwave Radiometer (DMR) observation of the cosmic microwave background (CMB) fluctuations [11,12]: (1) The density fluctuation amplitude is observed to be $\delta \simeq 10^{-5}$. (2) The value of the spectral index derived from the first year of COBE data is $n_s = 1.1 \pm 0.5$. (The COBE two year results are also available [13]. However, different statistical methods used in analyzing the data lead to different bounds on the spectral index [14]. For the purposes of this paper, we will take $n_s \geq 0.6$.) For models in which the field can be taken to be initially at the maximum of the potential, there is an additional consistency constraint. If the maximum of the potential is an unstable equilibrium, a field sitting at the maximum will be driven off by quantum fluctuations of amplitude ϕ_q, where ϕ_q on the scale of a horizon size is given by:

$$\phi_q = \frac{H}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{8\pi}{3m_{Pl}^2}V(0)}. \tag{2.12}$$

If $N(\phi_q) < 60$, the universe will not inflate sufficiently, so we have the constraint for successful inflation that

$$(\phi_q/\phi_{60}) < 1. \tag{2.13}$$

In the case of models, such as natural inflation, in which the initial conditions are randomly selected, this constraint takes a somewhat different form.

B. Natural inflation

"Natural" inflationary theories [15,16] use a pseudo Nambu-Goldstone boson to drive the inflationary expansion. The basic scenario consists of the following: A spontaneous symmetry breaking phase transition occurs at a scale v, and temperature $T \simeq v$. For definiteness, consider the simple case of symmetry breaking involving a single complex scalar field ϕ, with a potential

$$V(\phi) = \lambda \left[\phi^* \phi - v^2 \right]^2, \tag{2.14}$$

which is symmetric under a global U(1) transformation $\phi \to e^{i\alpha}\phi$. At the minimum of the potential $V(\phi)$, we can parameterize the scalar field as $\phi = \sigma \exp[i\theta/v]$. The radial field σ has a mass $M_\sigma^2 \simeq \lambda v^2$. The field θ is a Nambu-Goldstone boson, and is massless at tree level. If the U(1) symmetry of the potential $V(\theta)$ is preserved by the rest of the Lagrangian, θ remains massless with loop corrections. But if the U(1) is broken by other terms in the Lagrangian, θ acquires a potential $V_1(\theta)$ from loop corrections, and acquires a nonzero mass. Then θ is called a *pseudo Nambu-Goldstone boson* (PNGB). (Models with explicitly broken global U(1) symmetries have been discussed in the literature [16,17]. In this paper, we consider an inflationary potential created by gauge boson loop effects in a Lagrangian with an explicitly broken SO(3) symmetry.) Assuming the mass of θ is much less than that of the radial mode, $M_\theta^2 \ll M_\sigma^2$, the field θ is effectively massless near the original symmetry breaking scale $T \simeq v$. As the temperature of the universe decreases, $T \ll M_\sigma$, excitations of the heavy σ field are damped, so that we can take $\sigma = v = const$. The only remaining degree of freedom is θ, and we can parameterize ϕ as:

$$\phi = v e^{i\theta/v}. \tag{2.15}$$

At temperatures $T \gg M_\theta$, the effective potential $V_1(\theta)$ is negligible. When the universe cools to $T \simeq M_\theta$, $V_1(\theta)$ becomes important [15]. The field θ rolls down the potential to its minimum, resulting in inflationary expansion during the period in which the energy density

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of the universe is dominated by vacuum energy. Natural inflationary models typically assume a potential for the PNGB of the form

$$V_1(\theta) = \Lambda^4 \left[1 + \cos\left(\frac{\theta}{v}\right) \right], \qquad (2.16)$$

where v is the original symmetry breaking scale, and Λ is an independent energy scale characterizing the temperature at which the potential $V_1(\theta)$ becomes significant.

Note that in natural inflation we *cannot* assume an initial field value near the quantum fluctuation limit θ_q, and the consistency condition (2.13) must be modified. As the universe cools to $T \simeq \Lambda$, we expect the field θ and its derivative $\dot{\theta}$ to take on different values in different regions of the universe; we assume that the field is, to a good approximation, uniform within any pre-inflation horizon volume. The universe just prior to inflation therefore consists of a large number of causally disconnected regions, each with independent initial conditions for θ and $\dot{\theta}$. Each independent region will inflate a different amount, or perhaps not at all, depending on the conditions within that region. In a successful model for inflation, the *post*-inflation universe is strongly dominated by regions in which $N(\theta) \geq 60$. It can be shown that the initial value of $\dot{\theta}$ does not significantly affect the number of e-folds of inflation [18], and hence we consider here only the upper limit $\theta \leq \theta_{60}$. Consider a pre-inflation horizon volume V_0 and initial field value θ; during inflation, this region expands to a volume $V = V_0 \exp[3N(\theta)]$. The fraction of the volume of the post-inflation universe for which $N(\theta) \geq 60$ is then [16]

$$F(N \geq 60) = 1 - \left(\int_{\theta_{60}}^{\theta_{min}} \exp[3N(\theta)] \, d\theta \bigg/ \int_{\theta_q}^{\theta_{min}} \exp[3N(\theta)] \, d\theta \right), \qquad (2.17)$$

where θ_{min} is the minimum of the potential, and $N(\theta > \theta_f) \equiv 0$. Here a cutoff $\theta_q \equiv H/2\pi$, the magnitude of quantum fluctuations on the scale of a horizon size, has been introduced as the lower limit for the field value. A successful inflationary theory has the resulting characteristic that

$$\Pi(\theta_{60}) \equiv \int_{\theta_{60}}^{\theta_{min}} \exp[3N(\theta)] \, d\theta \bigg/ \int_{\theta_q}^{\theta_{min}} \exp[3N(\theta)] \, d\theta \ll 1. \qquad (2.18)$$

This condition can be satisfied even for models in which θ_{60} is constrained to be extremely small, as long as $\theta_q \ll \theta_{60}$, and $N(\theta_q) \gg 60$. The integral (2.18) is in most cases difficult to calculate. However, a very rough limit on its magnitude can be taken to be:

$$\Pi < \frac{\exp\left[3N\left(\theta_{60}\right)\right]}{\exp\left[3N\left(\theta_q\right)\right]}. \tag{2.19}$$

In the cases considered in this paper, natural inflation models which satisfy other observational constraints are also characterized by $\Pi \ll 1$, and this consistency condition does not provide a significant constraint.

III. INFLATIONARY CONSTRAINTS FOR GENERAL POTENTIALS

Scalar field potentials arising from spontaneous symmetry breaking can in general be characterized by the presence of a "false" vacuum, an unstable or metastable equilibrium with nonzero vacuum energy density, and a physical vacuum, for which the classical expectation value of the scalar field is nonzero. At the physical vacuum, the potential has a stable minimum where the vacuum energy density is defined to vanish. In this paper, we consider potentials for "new" inflation, where the false vacuum is an *unstable* equilibrium, and inflation takes place during a period of slow-roll. Take a potential $V(\phi)$ described by a symmetry breaking scale v and a vacuum energy density Λ^4:

$$V(\phi) = \Lambda^4 f\left(\frac{\phi}{v}\right). \tag{3.1}$$

We take the first derivative of the potential to be zero at the origin and the minimum to be at $\phi = \phi_{min} \sim v$:

$$f'(0) = f'(\phi_{min}) = 0,$$
$$f(0) = 1, \ f(\phi_{min}) = 0. \tag{3.2}$$

The first order inflationary parameter $\epsilon(\phi)$ is given by

$$\epsilon(\phi) \equiv \frac{m_{Pl}^2}{16\pi}\left(\frac{V'(\phi)}{V(\phi)}\right)^2$$

$$= \frac{1}{16\pi} \left(\frac{m_{Pl}}{v}\right)^2 \left(\frac{f'\left(\phi/v\right)}{f\left(\phi/v\right)}\right)^2. \tag{3.3}$$

An inflationary phase is characterized by $\epsilon < 1$: here $\epsilon\left(\phi = 0\right) = 0$ by construction. If $\epsilon\left(\phi\right)$ is everywhere increasing on the range $0 \leq \phi < \phi_{min}$, there is a unique field value ϕ_f at which inflation ends, where $\epsilon\left(\phi_f\right) \equiv 1$ and $\epsilon\left(\phi < \phi_f\right) < 1$. We are particularly interested in cases where the symmetry breaking takes place well below the Planck scale, $v \ll m_{Pl}$. Noting from (3.3) that $\epsilon \propto \left(m_{Pl}/v\right)^2$, the field value ϕ_f at which inflation ends is small for $v \ll m_{Pl}$, and we need only consider the behavior of the potential near the origin. We perform a Taylor expansion of $V\left(\phi\right)$ about the origin:

$$V\left(\phi\right) = V\left(0\right) + \frac{1}{m!}\frac{d^m V}{d\phi^m}\bigg|_{\phi=0} \phi^m + \cdots, \tag{3.4}$$

where $V'\left(0\right) \equiv 0$, and m is the order of the lowest non-vanishing derivative at the origin. (In Section IV we consider a case for which the potential does not have a well-defined Taylor expansion about the origin, but for the moment we assume that the series above is convergent). For cases in which the origin is a maximum of the potential, m must be even, and $d^m V/d\phi^m < 0$. For m odd, the origin is at a saddle point, and we can define the positive ϕ direction to be such that $d^m V/d\phi^m < 0$. It is to be expected that for most potentials arising from spontaneous symmetry breaking, inflation will take place near an unstable maximum and m will be even, but this is an unnecessarily strict condition for the purpose of a general analysis. The potential can be written in the form

$$V\left(\phi\right) = \Lambda^4 \left[1 - \frac{1}{m}\left(\frac{\phi}{\mu}\right)^m + \cdots\right], \tag{3.5}$$

so that for $\left(\phi/\mu\right) \ll 1$, the potential is dominated by terms of order $\left(\phi/\mu\right)^m$. The vacuum energy density is $\Lambda^4 \equiv V\left(0\right)$, and μ is an effective symmetry breaking scale defined by

$$\mu \equiv \left(\frac{\left(m-1\right)! V\left(\phi\right)}{\left|d^m V/d\phi^m\right|}\right)^{1/m}\bigg|_{\phi=0} = v\left(\frac{\left(m-1\right)! f\left(x\right)}{\left|d^m f/dx^m\right|}\right)^{1/m}\bigg|_{x=0}, \tag{3.6}$$

We wish to evaluate the characteristics of potentials of this general form when constrained by cosmological observations. The program is: (1) From $\epsilon\left(\phi_f\right) = 1$, calculate ϕ_f. (2) With ϕ_f

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determined, calculate $N(\phi)$ and determine ϕ_{60}. (3) From ϕ_{60}, calculate δ, which constrains Λ, and n_s, which constrains μ. (4) From Λ, calculate ϕ_q and verify $(\phi_q/\phi_{60}) < 1$. The cases $m = 2$ and $m > 2$ exhibit strikingly different behavior, and we consider each separately.

A. Quadratic potentials

Given a potential with a nonzero second derivative at the origin, $m = 2$, we take, for $(\phi/\mu) \ll 1$

$$V(\phi) \simeq \Lambda^4 \left[1 - \frac{1}{2} \left(\frac{\phi}{\mu} \right)^2 \right], \tag{3.7}$$

where the effective symmetry breaking scale is

$$\mu \equiv \left| \frac{V(0)}{V''(0)} \right|^{1/2}. \tag{3.8}$$

For example, if we take a standard potential for spontaneous symmetry breaking, $V(\phi) = \lambda \left[\phi^2 - v^2 \right]^2$, the effective symmetry breaking scale is given by $\mu = v/2$, and the vacuum energy density is $\Lambda^4 = \lambda v^4$. Inflation occurs for field values $\phi < \phi_f$, where

$$\epsilon(\phi_f) = \frac{1}{16\pi} \left(\frac{m_{Pl}}{\mu} \right)^2 \left(\frac{(\phi_f/\mu)}{1 - (1/2)(\phi_f/\mu)^2} \right)^2 \simeq \frac{1}{16\pi} \left(\frac{m_{Pl}}{\mu} \right)^2 \left(\frac{\phi_f}{\mu} \right)^2 \equiv 1. \tag{3.9}$$

We then have an expression for ϕ_f as a function of the scale μ:

$$\left(\frac{\phi_f}{\mu} \right) = \sqrt{16\pi} \left(\frac{\mu}{m_{Pl}} \right), \tag{3.10}$$

which confirms the consistency of the approximation $(\phi/\mu) \ll 1$ for $\mu \ll m_{Pl}$. The number of e-folds $N(\phi)$ is given by

$$\begin{aligned} N(\phi) &= -8\pi \left(\frac{\mu}{m_{Pl}} \right)^2 \int_{\phi_f/\mu}^{\phi/\mu} \frac{1 - x^2/2}{x} dx \\ &\simeq 8\pi \left(\frac{\mu}{m_{Pl}} \right)^2 \ln(\phi_f/\phi). \end{aligned} \tag{3.11}$$

Using the value for ϕ_f in (3.10), the upper limit ϕ_{60} on the initial field value is

$$\left(\frac{\phi_{60}}{\mu} \right) = \sqrt{16\pi} \left(\frac{\mu}{m_{Pl}} \right) \exp \left[-\frac{15}{2\pi} \left(\frac{m_{Pl}}{\mu} \right)^2 \right], \tag{3.12}$$

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decaying exponentially with decrease in the scale μ. Scalar density fluctuations are generated with an amplitude

$$
\begin{aligned}
\delta &= \sqrt{\frac{2}{\pi}} \left(\frac{\Lambda^2 \mu}{m_{Pl}^3} \right) \frac{\left[1 - (1/2) \left(\phi_{60}/\mu \right)^2 \right]^{3/2}}{(\phi_{60}/\mu)} \\
&\simeq \sqrt{\frac{2}{\pi}} \left(\frac{\mu}{m_{Pl}} \right)^3 \left(\frac{\Lambda}{\mu} \right)^2 \left(\frac{\mu}{\phi_{60}} \right).
\end{aligned}
\tag{3.13}
$$

Substituting ϕ_{60} from (3.12),

$$
\delta = \sqrt{\frac{1}{8\pi^2}} \left(\frac{\mu}{m_{Pl}} \right)^2 \left(\frac{\Lambda}{\mu} \right)^2 \exp \left[\frac{15}{2\pi} \left(\frac{m_{Pl}}{\mu} \right)^2 \right],
\tag{3.14}
$$

with the result that the density fluctuation amplitude grows exponentially with decreasing scale μ. In order to remain consistent with the COBE DMR result $\delta \simeq 10^{-5}$, we must take $\Lambda \equiv V(0)^{(1/4)}$ to be

$$
\begin{aligned}
\left(\frac{\Lambda}{\mu} \right)^2 &= \delta \sqrt{\frac{\pi}{2}} \left(\frac{m_{Pl}}{\mu} \right)^3 \left(\frac{\phi_{60}}{\mu} \right) \\
&= \pi \sqrt{8} \, \delta \left(\frac{m_{Pl}}{\mu} \right)^2 \exp \left[-\frac{15}{2\pi} \left(\frac{m_{Pl}}{\mu} \right)^2 \right].
\end{aligned}
\tag{3.15}
$$

This illustrates the "fine-tuning" problem for inflationary models of this form – the vacuum energy density Λ is constrained to decay exponentially as (μ/m_{Pl}) decreases. For the potential $V(\phi) = \lambda \left[\phi^2 - v^2 \right]^2$, we have $(\Lambda/\mu)^2 = \lambda^{1/2}$, and the constraint (3.15) forces the scalar coupling λ to extremely small values. In the sense that the parameter Λ depends exponentially on the symmetry breaking scale, fine-tuning is seen to be generic to potentials of this type. The tensor fluctuation amplitude a_T is:

$$
\begin{aligned}
a_T &= \frac{2}{\sqrt{3\pi}} \frac{[V(\phi_{60})]^{1/2}}{m_{Pl}^2} = \frac{2}{\sqrt{3\pi}} \left(\frac{\Lambda}{\mu} \right)^2 \left(\frac{\mu}{m_{Pl}} \right)^2 \\
&= 2\delta \sqrt{\frac{8\pi}{3}} \exp \left[-\frac{15}{2\pi} \left(\frac{m_{Pl}}{\mu} \right)^2 \right],
\end{aligned}
\tag{3.16}
$$

and tensor fluctuations are strongly suppressed at low scale. The magnitude of quantum fluctuations is given by:

$$
\left(\frac{\phi_q}{\mu} \right) = \sqrt{\frac{2}{3\pi}} \left(\frac{\mu}{m_{Pl}} \right) \left(\frac{\Lambda}{\mu} \right)^2
$$

$$= \sqrt{\frac{16\pi}{3}} \left(\frac{m_{Pl}}{\mu}\right) \exp\left[-\frac{15}{2\pi}\left(\frac{m_{Pl}}{\mu}\right)^2\right]. \tag{3.17}$$

From (3.12) we have the relationship

$$\left(\frac{\phi_q}{\phi_{60}}\right) = \frac{\delta}{\sqrt{3}}\left(\frac{m_{Pl}}{\mu}\right)^2, \tag{3.18}$$

and the consistency condition $(\phi_q/\phi_{60}) < 1$ places a lower bound on the scale μ:

$$\left(\frac{\mu}{m_{Pl}}\right) \gtrsim \delta^{(1/2)} \simeq 3 \times 10^{-3}. \tag{3.19}$$

A much stricter lower limit on (μ/m_{Pl}) can, however, be derived from the COBE limit on the scalar spectral index $n_s \geq 0.6$, where, for $(\phi_{60}/\mu) \ll 1$,

$$\begin{aligned} n_s &= 1 - 4\epsilon\left(\phi_{60}\right) + 2\eta\left(\phi_{60}\right) \\ &= 1 - \frac{3m_{Pl}^2}{8\pi}\left(\frac{V'\left(\phi_{60}\right)}{V\left(\phi_{60}\right)}\right)^2 + \frac{m_{Pl}^2}{4\pi}\left(\frac{V''\left(\phi_{60}\right)}{V\left(\phi_{60}\right)}\right) \\ &\simeq 1 + \frac{m_{Pl}^2}{4\pi}\left(\frac{V''\left(\phi_{60}\right)}{V\left(\phi_{60}\right)}\right). \end{aligned} \tag{3.20}$$

For a potential of the form (3.7)

$$\begin{aligned} 1 + \frac{m_{Pl}^2}{4\pi}\left(\frac{V''\left(\phi_{60}\right)}{V\left(\phi_{60}\right)}\right) &= 1 - \frac{1}{4\pi}\left(\frac{m_{Pl}}{\mu}\right)^2 \frac{1}{1 - (1/2)\left(\phi_{60}/\mu\right)^2} \\ &\simeq 1 - \frac{1}{4\pi}\left(\frac{m_{Pl}}{\mu}\right)^2, \end{aligned} \tag{3.21}$$

and taking $n_s \geq 0.6$, we obtain the lower limit

$$\left(\frac{\mu}{m_{Pl}}\right) \gtrsim 0.4. \tag{3.22}$$

This is a problematic result, however, since it precludes inflation driven by symmetry breaking near, for instance, the grand unified scale, $m_{GUT} \simeq 10^{-3} m_{Pl}$. It would be desirable to find a class of inflationary potentials which satisfy observational constraints for scales $\mu \ll m_{Pl}$. In the next section, we show that potentials of the form (3.5) with $m > 2$ satisfy observational constraints at *arbitrarily low* scales μ, removing the substantial restriction presented by the lower limit in equation (3.22).

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B. Higher order potentials

Now consider a potential for which the second derivative vanishes at the origin, $V''(0) = 0$:

$$\left.\frac{d^n V}{d\phi^n}\right|_{\phi=0} = 0 \quad (n < m), \tag{3.23}$$

where $m > 2$. For small (ϕ/μ), we can write the potential as

$$V(\phi) \simeq \Lambda^4 \left[1 - \frac{1}{m}\left(\frac{\phi}{\mu}\right)^m\right]. \tag{3.24}$$

The effective symmetry breaking scale μ is given by equation (3.6). We solve for the dependence of the inflationary constraints on the parameters μ and Λ following the same procedure as for the $m = 2$ case. The first order inflationary parameter ϵ is given by

$$\epsilon(\phi) = \frac{1}{16\pi}\left(\frac{m_{Pl}}{\mu}\right)^2 \left(\frac{(\phi/\mu)^{(m-1)}}{1 - (1/m)(\phi/\mu)^m}\right)^2 \simeq \frac{1}{16\pi}\left(\frac{m_{Pl}}{\mu}\right)^2 \left(\frac{\phi}{\mu}\right)^{2(m-1)}. \tag{3.25}$$

Taking $\epsilon(\phi_f) \equiv 1$, we have for ϕ_f

$$\left(\frac{\phi_f}{\mu}\right) = \left[\sqrt{16\pi}\left(\frac{\mu}{m_{Pl}}\right)\right]^{1/(m-1)}. \tag{3.26}$$

The number of e-folds $N(\phi)$ is [19]

$$N(\phi) = -8\pi \left(\frac{\mu}{m_{Pl}}\right)^2 \int_{\phi_f/\mu}^{\phi/\mu} \frac{1 - x^m/m}{x^{m-1}}dx$$
$$\simeq 8\pi \left(\frac{\mu}{m_{Pl}}\right)^2 \left(\frac{1}{m-2}\right)\left[\left(\frac{\mu}{\phi}\right)^{m-2} - \left(\frac{\mu}{\phi_f}\right)^{m-2}\right]. \tag{3.27}$$

Substituting (3.26) for ϕ_f, we then have for ϕ_{60}:

$$\left(\frac{\phi_{60}}{\mu}\right) = \left\{\frac{15(m-2)}{2\pi}\left(\frac{m_{Pl}}{\mu}\right)^2 + \left[\frac{1}{\sqrt{16\pi}}\left(\frac{m_{Pl}}{\mu}\right)\right]^{(m-2)/(m-1)}\right\}^{-1/(m-2)}. \tag{3.28}$$

Since $(m-2)/(m-1) < 1$, the $(m_{Pl}/\mu)^2$ term dominates, and we have the result that ϕ_{60} is to a good approximation *independent* of ϕ_f for $\mu \ll m_{Pl}$ [20]:

$$\left(\frac{\phi_{60}}{\mu}\right) \simeq \left[\frac{2\pi}{15(m-2)}\left(\frac{\mu}{m_{Pl}}\right)^2\right]^{1/(m-2)}. \tag{3.29}$$

This independence will be important when we consider the consistency of the slow-roll approximation. In this case, ϕ_{60} decreases as a power law in (μ/m_{Pl}) rather than exponentially, as in (3.12). The density fluctuation amplitude δ is

$$
\begin{aligned}
\delta &= \sqrt{\frac{2}{\pi}} \left(\frac{\Lambda^2 \mu}{m_{Pl}^3} \right) \frac{[1 - (1/m)(\phi_{60}/\mu)^m]^{3/2}}{(\phi_{60}/\mu)^{m-1}} \\
&\simeq \sqrt{\frac{2}{\pi}} \left(\frac{\mu}{m_{Pl}} \right)^3 \left(\frac{\Lambda}{\mu} \right)^2 \left(\frac{\mu}{\phi_{60}} \right)^{m-1} .
\end{aligned}
\tag{3.30}
$$

Substituting ϕ_{60} from (3.29), we have

$$
\delta = \sqrt{\frac{2}{\pi}} \left(\frac{15(m-2)}{2\pi} \right)^{(m-1)/(m-2)} \left(\frac{\Lambda}{\mu} \right)^2 \left(\frac{\mu}{m_{Pl}} \right)^{(m-4)/(m-2)} .
\tag{3.31}
$$

The dependence of the density fluctuation amplitude δ on μ is then power law, instead of expontential as in the $m = 2$ case (3.14). For the case $m = 4$, which will be of interest in the context of a specific model, the density fluctuation amplitude is independent of (μ/m_{Pl}). For $m > 4$, δ decreases with decreasing (μ/m_{Pl}) – production of density fluctuations is suppressed at low scale. The COBE DMR constraint on the vacuum energy density Λ is then

$$
\left(\frac{\Lambda}{\mu} \right)^2 = \delta \sqrt{\frac{\pi}{2}} \left(\frac{2\pi}{15(m-2)} \right)^{(m-1)/(m-2)} \left(\frac{m_{Pl}}{\mu} \right)^{(m-4)/(m-2)} ,
\tag{3.32}
$$

and the constraint requires no fine-tuning of constants. The amplitude of tensor fluctuations is

$$
\begin{aligned}
a_T &= \frac{2}{\sqrt{3\pi}} \left(\frac{\Lambda}{\mu} \right)^2 \left(\frac{\mu}{m_{Pl}} \right)^2 \\
&= \delta \sqrt{\frac{2}{3}} \left(\frac{2\pi}{15(m-2)} \right)^{(m-1)/(m-2)} \left(\frac{\mu}{m_{Pl}} \right)^{m/(m-2)} ,
\end{aligned}
\tag{3.33}
$$

and tensor fluctuations are small for $\mu \ll m_{Pl}$. The quantum fluctuation amplitude ϕ_q is given by:

$$
\begin{aligned}
\left(\frac{\phi_q}{\mu} \right) &= \sqrt{\frac{2}{3\pi}} \left(\frac{\mu}{m_{Pl}} \right) \left(\frac{\Lambda}{\mu} \right)^2 \\
&= \frac{\delta}{\sqrt{3}} \left(\frac{2\pi}{15(m-2)} \right)^{(m-1)/(m-2)} \left(\frac{\mu}{m_{Pl}} \right)^{2/(m-2)} ,
\end{aligned}
\tag{3.34}
$$

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and the condition $(\phi_q/\phi_{60}) < 1$ is satisfied independent of (μ/m_{Pl}):

$$\left(\frac{\phi_q}{\phi_{60}}\right) = \frac{2\pi\delta}{15\sqrt{3}(m-2)}. \tag{3.35}$$

The number of e-folds becomes very large at the quantum fluctuation amplitude

$$N\left(\phi_q\right) = 8\pi \left(\frac{15}{2\pi}\right)^{m-1} \left(\frac{(m-2)\sqrt{3}}{\delta}\right)^{m-2} \simeq 10^{5(m-2)}. \tag{3.36}$$

Even for randomly selected initial conditions, as in a PNGB model, we see that the volume of the post-inflation universe is vastly dominated by sufficiently inflated regions, where we can take a rough upper limit on Π in equation (2.18) to be

$$\Pi < \exp\left[180 - 3N\left(\phi_q\right)\right] \simeq \exp\left[-10^{5(m-2)}\right] \ll 1. \tag{3.37}$$

The scalar spectral index is given by:

$$\begin{aligned} n_s &\simeq 1 + \frac{m_{Pl}^2}{4\pi} \frac{V''\left(\phi_{60}\right)}{V\left(\phi_{60}\right)} \\ &\simeq 1 - \frac{m-1}{4\pi} \left(\frac{m_{Pl}}{\mu}\right)^2 \left(\frac{\phi_{60}}{\mu}\right)^{m-2} \\ &= 1 - \left(\frac{1}{30}\right) \frac{m-1}{m-2}, \end{aligned} \tag{3.38}$$

and we have the rather surprising result that for any $m > 2$, the scalar spectral index is independent of any characteristic of the potential except the order of the lowest non-vanishing derivative at the origin. The constraint from COBE is automatically met, with $0.93 < n_s < 0.97$ for all values of m.

One apparent difficulty with this class of potentials, however, is that the second order slow-roll parameter $|\eta|$ becomes large for $\phi \ll \phi_f$, so that the slow-roll approximation is invalid over much of the range at which inflation is taking place. Inflation ends at the field value ϕ_f given by (3.26), but *slow-roll* ends at

$$\begin{aligned} |\eta\left(\phi\right)| &\simeq \frac{m_{Pl}^2}{8\pi} \left|\frac{V''\left(\phi\right)}{V\left(\phi\right)}\right| = 1 \\ \left(\frac{\phi}{\mu}\right) &= \left[\frac{8\pi}{m-1} \left(\frac{\mu}{m_{Pl}}\right)^2\right]^{1/(m-2)} \ll \left(\frac{\phi_f}{\mu}\right). \end{aligned} \tag{3.39}$$

However, from equation (3.29), ϕ_{60} is *independent* of ϕ_f, so that the breakdown of slow-roll has no effect, as long as slow-roll is valid at the initial field value, $|\eta(\phi_{60})| < 1$. If we define ϕ_{60} to be 60 e-folds before the end of slow-roll as defined in (3.39), instead of the end of inflation proper, we have, using (3.27) for $N(\phi)$,

$$\left(\frac{\phi_{60}}{\mu}\right) \simeq \left[\frac{2\pi}{15(m-2)}\left(\frac{\mu}{m_{Pl}}\right)^2\right]^{1/(m-2)}\left(1+\frac{m-1}{60(m-2)}\right)^{-1/(m-2)}, \tag{3.40}$$

which is a small correction to equation (3.29).

C. Summary

In this section we summarize the results for potentials of the form (3.5) for the cases $m = 2$ and $m > 2$. For $m = 2$,

$$\delta = \sqrt{\frac{1}{8\pi^2}\left(\frac{\mu}{m_{Pl}}\right)^2\left(\frac{\Lambda}{\mu}\right)^2}\exp\left[\frac{15}{2\pi}\left(\frac{m_{Pl}}{\mu}\right)^2\right],$$
$$n_s = 1 - \frac{1}{4\pi}\left(\frac{m_{Pl}}{\mu}\right)^2,$$
$$\left(\frac{\phi_q}{\phi_{60}}\right) = \frac{\delta}{\sqrt{3}}\left(\frac{m_{Pl}}{\mu}\right)^2$$
$$a_T = 2\delta\sqrt{\frac{8\pi}{3}}\exp\left[-\frac{15}{2\pi}\left(\frac{m_{Pl}}{\mu}\right)^2\right]. \tag{3.41}$$

Here the COBE measurement of the scalar spectral index $n_s \geq 0.6$ forces the effective symmetry breaking scale to be near the Planck scale, $(\mu/m_{Pl}) > 0.4$, and inflation from symmetry breaking at low scales is inconsistent with observational constraints. For $m > 2$, the corresponding result is:

$$\delta = \sqrt{\frac{2}{\pi}}\left(\frac{15(m-2)}{2\pi}\right)^{(m-1)/(m-2)}\left(\frac{\Lambda}{\mu}\right)^2\left(\frac{\mu}{m_{Pl}}\right)^{(m-4)/(m-2)}$$
$$n_s = 1 - \frac{1}{30}\left(\frac{m-1}{m-2}\right)$$
$$\left(\frac{\phi_q}{\phi_{60}}\right) = \frac{2\pi\delta}{15\sqrt{3}(m-2)}$$
$$a_T = \delta\sqrt{\frac{2}{3}}\left(\frac{2\pi}{15(m-2)}\right)^{(m-1)/(m-2)}\left(\frac{\mu}{m_{Pl}}\right)^{m/(m-2)}. \tag{3.42}$$

The scalar spectral index n_s is independent of any characteristic of the potential except m. For the case $m = 4$, the density fluctuation amplitude is independent of (μ/m_{Pl}), and inflation can take place successfully at an arbitrary symmetry breaking scale. In Section IV, we illustrate this behavior within the context of a specific model, in which the potential is created by loop effects in a Lagrangian with an explicitly broken SO(3) symmetry.

It should be noted that it is not strictly necessary that $V''(0)$ vanish for inflation to be characterized by (3.42) for some range of effective symmetry breaking scales μ. It is sufficient that the second derivative at the origin be small relative to some derivative of order $m > 2$. If there is a range of ϕ_{60} such that

$$\frac{1}{2}\left|\frac{V''(0)}{V(0)}\right|\phi_{60}^2 \ll \frac{1}{m!}\frac{1}{V(0)}\left|\frac{d^m V}{d\phi^m}\right|_{\phi=0}\phi_{60}^m \equiv \frac{1}{m}\left(\frac{\phi_{60}}{\mu}\right)^m, \tag{3.43}$$

then the potential is still be dominated by terms of order ϕ^m for $\phi \geq \phi_{60}$, and $V(\phi)$ is of the form (3.5) to a good approximation for field values of physical interest. Taking ϕ_{60} to be approximately of the form (3.29), we have

$$\frac{2\pi}{15(m-2)}\left(\frac{\mu}{m_{Pl}}\right)^2 > \mu^2\left(\frac{m}{2}\right)\left|\frac{V''(0)}{V(0)}\right| = 4\pi m\left(\frac{\mu}{m_{Pl}}\right)^2|\eta(0)|. \tag{3.44}$$

We then have a constraint on the value of the second order slow-roll parameter η at the origin

$$|\eta(0)| < \frac{1}{30m(m-2)}. \tag{3.45}$$

If we write $V(\phi)$ in terms of a dimensionless function f as in (3.1), the parameter $\eta(0)$ is given by

$$\eta(0) = \frac{1}{8\pi}\left(\frac{m_{Pl}}{v}\right)^2\frac{f''(0)}{f(0)} \propto \left(\frac{m_{Pl}}{v}\right)^2, \tag{3.46}$$

resulting in a lower limit on the symmetry breaking scale v,

$$\left(\frac{v}{m_{Pl}}\right)^2 < \frac{15m(m-2)}{4\pi}\left|\frac{f''(0)}{f(0)}\right|, \tag{3.47}$$

which depends on the particular form of the potential. This is illustrated in the context of a specific model in Section IV.

IV. INFLATION FROM SO(3) PSEUDO NAMBU-GOLDSTONE BOSONS

It is not immediately clear that potentials with $m > 2$ can be generated by spontaneous symmetry breaking, since such symmetry breaking is, at least at tree level, created by scalar mass terms. A Lagrangian of the generic form

$$\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - \frac{1}{2}\mu^2 \phi^2 - \frac{\lambda}{4}\phi^4 \tag{4.1}$$

only exhibits spontaneous symmetry breaking at tree level for $\mu^2 < 0$, and the potential is automatically dominated by quadratic terms near $\phi = 0$. One physically well motivated possibility for overcoming this difficulty is a model involving scalar particles which are massless at tree level, but which acquire mass through radiative corrections. The original models for new inflation [7,8] are of this type, using Coleman-Weinberg symmetry breaking to create the inflationary potential, with $m = 4$. Natural inflation models, using pseudo Nambu-Goldstone bosons to drive inflation, also belong to this category. Here we consider a natural inflation model in which the PNGB potential is created by loop effects from gauge bosons.

A. Pseudo Nambu-Goldstone bosons from gauge boson loops

Take a scalar particle Lagrangian which is invariant under some spontaneously broken gauge group G:

$$\mathcal{L} = (D_\mu \phi)^\dagger (D^\mu \phi) - \lambda \left[\phi^\dagger \phi - v^2\right]^2 - \frac{1}{4}Tr\left[F_{\mu\nu}F^{\mu\nu}\right]$$
$$D_\mu \equiv \partial_\mu - igQ_j A_\mu^j$$
$$F_{\mu\nu}^i \equiv \partial_\mu A_\nu^i - \partial_\nu A_\mu^i + gC_{ijk}A_\mu^j A_\nu^k, \tag{4.2}$$

where G has generators $\{Q_i\}$ with commutation relation $[Q_i, Q_j] = iC_{ijk}Q_k$. We take ϕ in a vector representation $\phi \equiv (\phi_1, \ldots \phi_n)$, which transforms under G as

$$\phi \to \exp\left[iQ_k \xi^k\right] \phi. \tag{4.3}$$

The product $\phi^\dagger \phi$ is then manifestly invariant under the group G. There is one gauge boson A_μ^i for each generator of the group G, with the gauge transformation law

$$A^i_\mu \to A^i_\mu + \frac{1}{g}\partial_\mu \xi^i + C_{ijk}A^j_\mu \xi^k. \tag{4.4}$$

For the Lagrangian (4.2), the group symmetry G is spontaneously broken when ϕ acquires a nonzero vacuum expectation value, $<\phi> = v$. We can parameterize ϕ in terms of a "shifted" field $<\sigma> = (0, \ldots, 0)$ and massless Nambu-Goldstone bosons ξ^i (*unitary gauge*):

$$\phi = \exp\left[iQ_k \xi^k\right][\sigma + \mathbf{v}], \tag{4.5}$$

where $\mathbf{v}$ is the vacuum expectation value $\mathbf{v} \equiv (v_1, \ldots, v_n)$, $\mathbf{v}^\dagger \mathbf{v} = v^2$. In the spontaneously broken vacuum, the gauge bosons acquire a mass

$$M^2_{ij} = g^2 \mathbf{v}^\dagger Q_i Q_j \mathbf{v}, \tag{4.6}$$

and the Nambu-Goldstone modes ξ^i which correspond to the broken generators of G are "eaten" to form longitudinal modes for the gauge bosons A_μ.

The gauge group, however, does not necessarily need to be the full symmetry group G. It is consistent to have gauge bosons which transform under some *subgroup* $\bar{G} \subset G$, with generators $\left\{\bar{Q}_i\right\} \subset \{Q_i\}$. If we take

$$D_\mu \equiv \partial_\mu - ig\bar{Q}_j A^j_\mu, \tag{4.7}$$

where $j = 1, \ldots, dim\left(\bar{G}\right)$, there are in general as many as $dim\left(G\right) - dim\left(\bar{G}\right)$ leftover massless Nambu-Goldstone bosons ξ. However, since the symmetry G of the scalar potential is not a symmetry of the entire Lagrangian, the leftover modes are PNGB's, and do not in general remain massless when radiative corrections are taken into account. This is reflected in the fact that the gauge boson mass matrix depends on the PNGB fields

$$M^2_{ij}(\xi) = g^2 \mathbf{v}^\dagger \exp\left[-iQ_k \xi^k\right] \bar{Q}_i \bar{Q}_j \exp\left[iQ_k \xi^k\right] \mathbf{v}, \tag{4.8}$$

where $i, j = 1, \ldots, dim\left(\bar{G}\right)$, and $k > dim\left(\bar{G}\right)$. Gauge boson loop effects generate a one-loop effective potential of the form [21]

$$V_1(\xi) = \frac{3}{64\pi^2}Tr\left\{\left[M^2(\xi)\right]^2 \ln\left[\frac{M^2(\xi)}{v^2}\right]\right\}. \tag{4.9}$$

We use an effective potential of this type to drive inflation. Note that the dependence of M_{ij}^2 on the modes ξ^k disappears if the commutator $\left[Q_k, \bar{Q}_j\right]$ vanishes for all $j = 1, \ldots, dim\left(\bar{G}\right)$, $k = dim\left(\bar{G}\right) + 1, \ldots, dim\left(G\right)$, so that the gauge group $\bar{G}$ must be a nontrivial subgroup of the scalar symmetry group G. In particular, if G is a direct product group, $G = G_1 \otimes G_2$, and $\bar{G} = G_1$, the dependence of V_1 on ξ vanishes.

B. PNGB's from an explicitly broken SO(3) symmetry

Take a Lagrangian with three real scalar fields ϕ_1, ϕ_2, ϕ_3 and an SO(3) symmetric potential

$$V\left(\phi\right) \equiv \lambda \left[\phi_1^2 + \phi_2^2 + \phi_3^2 - v^2\right]^2 . \tag{4.10}$$

It is convenient to parameterize the fields as a triplet

$$\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \\ \phi- \end{pmatrix}, \tag{4.11}$$

where

$$\phi^\pm \equiv \frac{1}{\sqrt{2}} \left(\phi_1 \pm \phi_2\right),$$
$$\phi^0 \equiv \phi_3. \tag{4.12}$$

In this basis, the generators of the SO(3) symmetry group can be taken to be

$$T_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$T_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & i \\ 0 & -i & 0 \end{pmatrix},$$

$$T_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \tag{4.13}$$

with commutation relation

$$[T_i, T_j] = -i\epsilon_{ijk}T_k, \tag{4.14}$$

where ϵ_{ijk} is the Levi-Civita tensor. Note that T_3 generates a U(1) subgroup of SO(3), where the $\phi^\pm$ fields are charged under the U(1) and the ϕ^0 field is neutral. We take the U(1) generated by T_3 to be the gauge group, with a Lagrangian of the form

$$\mathcal{L} = (D_\mu\phi)^\dagger (D^\mu\phi) - \lambda \left[\phi^\dagger\phi - v^2\right]^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu},$$
$$D_\mu \equiv \partial_\mu - igT_3A_\mu,$$
$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{4.15}$$

Here we a have a *single* gauge boson A_μ, transforming under the U(1) gauge symmetry generated by T_3, which explicitly breaks the SO(3) symmetry of the scalar potential. We choose the following general parameterization for ϕ:

$$\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \\ \phi- \end{pmatrix} = [\sigma + v] \begin{pmatrix} (1/\sqrt{2})e^{i(\alpha/v)}\sin(\theta/v) \\ \cos(\theta/v) \\ (1/\sqrt{2})e^{-i(\alpha/v)}\sin(\theta/v) \end{pmatrix}$$
$$= e^{iT_3(\alpha/v)} [\sigma + v] \begin{pmatrix} (1/\sqrt{2})\sin(\theta/v) \\ \cos(\theta/v) \\ (1/\sqrt{2})\sin(\theta/v) \end{pmatrix}, \tag{4.16}$$

which can be recognized as spherical coordinates in the basis (ϕ_1, ϕ_2, ϕ_3). The modes α and θ are the Nambu-Goldstone bosons. Note that although the SO(3) symmetry has three generators, there are only two Nambu-Goldstone bosons, since the SO(3) is spontaneously broken to SO(2), which has one generator. This residual symmetry corresponds to rotating the vacuum expectation vector $\mathbf{v}$ about itself, where

$$\mathbf{v} \equiv v e^{iT_3(\alpha/v)} \begin{pmatrix} (1/\sqrt{2})\sin(\theta/v) \\ \cos(\theta/v) \\ (1/\sqrt{2})\sin(\theta/v) \end{pmatrix}. \tag{4.17}$$

Note that the residual SO(2) symmetry of the vacuum *cannot* in general be identified with the U(1) gauge symmetry. In the spontaneously broken phase, the α mode is absorbed by the gauge boson A_μ, which acquires a mass

$$M^2 = g^2 \left(\mathbf{v}^\dagger T_3^2 \mathbf{v} \right)$$
$$= g^2 v^2 \sin^2 \left(\frac{\theta}{v} \right). \tag{4.18}$$

Loop effects generate a one-loop effective potential

$$V_1 \left(\theta \right) \equiv \frac{3}{64\pi^2} \left[M^2 \left(\theta \right) \right]^2 \ln \left[\frac{M^2 \left(\theta \right)}{v^2} \right]$$
$$= \frac{3v^4}{64\pi^2} g^4 \sin^4 \left(\frac{\theta}{v} \right) \ln \left[g^2 \sin^2 \left(\frac{\theta}{v} \right) \right], \tag{4.19}$$

which has a maximum at $\theta = 0$ and a minimum at $\theta = \pi v/2$, which is the physical vacuum. However, for a perturbative coupling, $g < 1$, the potential is negative at the physical vacuum, so we normalize to adjust the vacuum energy at the minimum to zero:

$$V \left(\theta \right) \equiv V_1 \left(\theta \right) - V_1 \left(\pi v/2 \right)$$
$$= \frac{3v^4}{64\pi^2} g^4 \left\{ \sin^4 \left(\frac{\theta}{v} \right) \ln \left[g^2 \sin^2 \left(\frac{\theta}{v} \right) \right] - \ln \left(g^2 \right) \right\} \tag{4.20}$$

We take this to be the inflationary potential, neglecting any effects due to contributions from a fermionic sector, which will be discussed later.

C. Inflation from SO(3) gauge potentials

For $(\theta/v) \ll 1$, the PNGB potential (4.20) becomes

$$V \left(\theta \right) \simeq \frac{3v^4}{64\pi^2} g^4 \left\{ \left(\frac{\theta}{v} \right)^4 \ln \left[g^2 \left(\frac{\theta}{v} \right)^2 \right] - \ln \left(g^2 \right) \right\}, \tag{4.21}$$

which is dominated by terms of order $(\theta/v)^4$, so we expect the inflationary constraints to be described by equation (3.42). However, the potential (4.21) does *not* have a well-defined Taylor expansion about the origin, since the fourth derivative, describing the PNGB self coupling, diverges logarithmically as $\theta \to 0$:

$$\left. \frac{d^m V}{d\theta^m} \right|_{\theta \to 0} = 0 \quad (m \leq 3),$$
$$\left. \frac{d^4 V}{d\theta^4} \right|_{\theta \to 0} \propto \ln \left(\frac{\theta}{v} \right) \to -\infty. \tag{4.22}$$

The origin of this behavior is the familiar infared divergence in the gauge boson propagator, since the U(1) gauge symmetry is unbroken at $\theta = 0$ and the gauge boson A_μ is massless. However, $V(\theta)$ does have a well-defined Taylor expansion about any *finite* field value θ_0, for which the gauge symmetry is broken and A_μ acquires a mass. For a field value $\theta \simeq \theta_0$, the potential is of the form

$$V(\theta) \simeq \frac{3v^4}{64\pi^2} g^4 \left[\left(\frac{\theta}{\mu}\right)^4 - \ln\left(g^2\right) \right], \tag{4.23}$$

where μ is a redefined mass scale which depends on the choice of θ_0. An appropriate value of θ_0 can be chosen as a function of the symmetry breaking scale v, and the results (3.42) are valid up to logarithmic corrections. The corrections can be determined iteratively. The potential can be written in the form:

$$V(\theta) = -\frac{3v^4}{64\pi^2} g^4 \ln\left(g^2\right) \left[1 - \frac{1}{4}\left(\frac{\theta}{\mu_0}\right)^4 \right] + \frac{3v^4}{256\pi^2} g^4 \left(\frac{\theta}{\mu_0}\right)^4 \ln\left[\frac{1}{2}\left(\frac{\theta}{\mu_0}\right)^2\right], \tag{4.24}$$

where $\mu_0 \equiv v/\sqrt{2}$. Expanding the logarithm about a field value $\theta = \theta_0$,

$$\ln\left[\frac{1}{2}\left(\frac{\theta}{\mu_0}\right)^2\right] = \ln\left[\frac{1}{2}\left(\frac{\theta_0}{\mu_0}\right)^2\right] - 2\sum_{n=1}^{\infty} \frac{1}{n}\left(1 - \frac{\theta}{\theta_0}\right)^n$$

$$\simeq \ln\left[\frac{1}{2}\left(\frac{\theta_0}{\mu_0}\right)^2\right] \quad (\theta \simeq \theta_0). \tag{4.25}$$

For $\theta \simeq \theta_0$, the potential is then of the form (3.5)

$$V(\theta) \simeq -\frac{3v^4}{64\pi^2} g^4 \ln\left(g^2\right) \left[1 - \frac{1}{4}\left(\frac{\theta}{\mu_0}\right)^4 \left(1 + \frac{\ln\left[(1/2)\left(\theta_0/\mu_0\right)^2\right]}{\ln\left(g^2\right)}\right) \right]$$

$$= \Lambda^4 \left[1 - \frac{1}{4}\left(\frac{\theta}{\mu_1}\right)^4 \right], \tag{4.26}$$

where the vacuum energy density is

$$\Lambda^4 \equiv -\frac{3v^4}{64\pi^2} g^4 \ln\left(g^2\right), \tag{4.27}$$

and the scale μ_1 is

$$\mu_1 \equiv \mu_0 \left(1 + \frac{\ln\left[(1/2)\left(\theta_0/\mu_0\right)^2\right]}{\ln\left(g^2\right)}\right)^{-1/4}. \tag{4.28}$$

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The scale μ_1 depends on the expansion parameter θ_0 and the gauge coupling g. Since all the quantities of physical interest are defined in terms of the field value $\theta = \theta_{60}$, we take the expansion parameter θ_0 to be the value of θ_{60} from equation (3.29) with m = 4:

$$\left(\frac{\theta_0}{\mu_0}\right) = \sqrt{\frac{\pi}{15}} \left(\frac{\mu_0}{m_{Pl}}\right). \tag{4.29}$$

Similarly, we determine the coupling constant to lowest order $g \simeq g_0$ from (3.32):

$$\left(\frac{\Lambda}{\mu_0}\right)^4 \equiv -\frac{3}{16\pi^2} g_0^4 \ln\left(g_0^2\right) = \frac{\pi}{2} \left(\frac{\pi}{15}\right)^3 \delta^2, \tag{4.30}$$

which is conveniently independent of μ_0. Note that the limit on Λ from the COBE value $\delta \simeq 10^{-5}$ serves in this context to constrain the gauge coupling g. To lowest order, the gauge coupling is independent of the symmetry breaking scale v, where

$$g_0^4 \ln\left(g_0^2\right) = -\frac{8}{3} \left(\frac{\pi^2}{15}\right)^3 \delta^2, \tag{4.31}$$

and we take the first order scale μ_1 to be

$$\mu_1 = \mu_0 \left(1 + \frac{\ln\left[(\pi/30)\left(\mu_0/m_{Pl}\right)^2\right]}{\ln\left(g_0^2\right)}\right)^{-1/4}. \tag{4.32}$$

We can then use (3.29) to obtain the first order correction to θ_{60}:

$$\left(\frac{\theta_{60}}{\mu_1}\right) = \sqrt{\frac{\pi}{15}} \left(\frac{\mu_1}{m_{Pl}}\right)$$

$$\Longrightarrow \left(\frac{\theta_{60}}{\mu_0}\right) = \sqrt{\frac{\pi}{15}} \left(\frac{\mu_0}{m_{Pl}}\right) \left(\frac{\mu_1}{\mu_0}\right)^2. \tag{4.33}$$

From (4.32), substituting $\mu_0 \equiv v/\sqrt{2}$,

$$\left(\frac{\theta_{60}}{v}\right) = \frac{1}{2}\sqrt{\frac{\pi}{15}} \left(\frac{v}{m_{Pl}}\right) \left(1 + \frac{\ln\left[(\pi/60)\left(v/m_{Pl}\right)^2\right]}{\ln\left(g_0^2\right)}\right)^{-1/2}, \tag{4.34}$$

where g_0 satisfies equation (4.31). We can also obtain a first order correction to the gauge coupling by using

$$\left(\frac{\Lambda}{\mu_1}\right)^4 = \left(\frac{\mu_0}{\mu_1}\right)^4 \left(\frac{\Lambda}{\mu_0}\right)^4 = -\frac{3}{16\pi^2} \left(\frac{\mu_0}{\mu_1}\right)^4 g^4 \ln\left(g^2\right)$$

$$= \frac{\pi}{2} \left(\frac{\pi}{15} \right)^3 \delta^2, \tag{4.35}$$

so that to first order, $g \simeq g_1$ satisfies

$$g_1^4 \ln \left(g_1^2 \right) = -\frac{8}{3} \left(\frac{\pi^2}{15} \right)^3 \delta^2 \left(\frac{\mu_1}{\mu_0} \right)^4$$

$$= -\frac{8}{3} \left(\frac{\pi^2}{15} \right)^3 \delta^2 \left(1 + \frac{\ln \left[(\pi/60) \left(v/m_{Pl} \right)^2 \right]}{\ln \left(g_0^2 \right)} \right)^{-1}, \tag{4.36}$$

and the COBE limited coupling constant does not require fine-tuning, with $g \simeq 10^{-3}$ for a wide range of symmetry breaking scales v. Since $(\theta_{60}/\mu_1) \propto (\mu_1/m_{Pl})$, the scalar spectral index is insensitive to the redefinition of scale, remaining independent of v:

$$n_s = 1 - \frac{3}{4\pi} \left(\frac{m_{Pl}}{\mu_1} \right)^2 \left(\frac{\theta_{60}}{\mu_1} \right)^2 = 1 - \frac{1}{20}$$

$$= 0.95. \tag{4.37}$$

The expansion (4.25) is valid only if the series

$$\sum_{n=1}^{\infty} \frac{1}{n} \left(1 - \frac{\theta}{\theta_0} \right)^n \tag{4.38}$$

converges at $\theta = \theta_{60}$ from (4.34). Taking θ_0 from (4.29),

$$\left(\frac{\theta_{60}}{\theta_0} \right) = \left(1 + \frac{\ln \left[(\pi/60) \left(v/m_{Pl} \right)^2 \right]}{\ln \left(g_0^2 \right)} \right)^{-1/2}, \tag{4.39}$$

which is less than one for all $v < m_{Pl}$, and (4.25) is consistent.

We then have the result that with a logarithmic divergence near the origin, the result (3.42) is approximately valid, with corrections derivable by an iterative solution:

$$\left(\frac{\theta_{60}}{v} \right) = \frac{1}{2} \sqrt{\frac{\pi}{15}} \left(\frac{v}{m_{Pl}} \right) \left(1 + \frac{\ln \left[(\pi/60) \left(v/m_{Pl} \right)^2 \right]}{\ln \left(g_0^2 \right)} \right)^{-1/2}$$

$$g^4 \ln \left(g^2 \right) = g_0^4 \ln \left(g_0^2 \right) \left(1 + \frac{\ln \left[(\pi/60) \left(v/m_{Pl} \right)^2 \right]}{\ln \left(g_0^2 \right)} \right)^{-1}$$

$$g_0^4 \ln \left(g_0^2 \right) \equiv -\frac{8}{3} \left(\frac{\pi^2}{15} \right)^3 \delta^2$$

$$n_s = 0.95 \tag{4.40}$$

Of particular note, the scalar spectral index n_s remains independent of the symmetry breaking scale v.

D. Inclusion of fermions

In this section we discuss the coupling of fermionic species to the scalars ϕ, and discuss the effects on the inflationary constraints which we obtained by considering only the effective potential generated by gauge boson loops. Clearly, we can preserve all of the previous results exactly if the Fermi sector is invariant under the full SO(3) symmetry group:

$$
\begin{aligned}
&\mathcal{L}_F = i\bar{\psi}\gamma^\mu D_\mu\psi - h\sum_{n=1}^{3}\left(\bar{\psi}_L\phi\psi_{nR} + \bar{\psi}_{nR}\phi^\dagger\psi_L\right), \\
&D_\mu \equiv \partial_\mu - igT_3A_\mu,
\end{aligned}
\tag{4.41}
$$

where the ψ_{nR} are singlet fields and ψ_L is a triplet

$$
\psi_L \equiv \begin{pmatrix} \psi_{1L} \\ \psi_{2L} \\ \psi_{3L} \end{pmatrix},
\tag{4.42}
$$

which transforms under the SO(3) as

$$
\psi_L \rightarrow \exp\left[iT_k\xi^k\right]\psi_L.
\tag{4.43}
$$

In this case the fermion mass is independent of the value of the PNGB mode θ, and there is no correction to the effective potential (4.20) except for the addition of a constant.

However, we can couple fermions which also explicitly break the SO(3) symmetry, as long as the Lagrangian respects the U(1) gauge symmetry. The simplest fermion Lagrangian of this type is of the form

$$
\mathcal{L}_F = i\bar{\psi}_1\gamma^\mu D_\mu\psi_1 + i\bar{\psi}_2\gamma^\mu\partial_\mu\psi_2 - h_1\left(\bar{\psi}_{1L}\phi^+\psi_{1R} + \bar{\psi}_{1R}\phi^-\psi_{1L}\right) - h_2\bar{\psi}_2\phi^0\psi_2,
\tag{4.44}
$$

where ψ_1 is charged under the U(1) gauge group and ψ_2 is neutral:

$$
\begin{aligned}
&\psi_{1L} \rightarrow e^{i\alpha/2}\psi_{1L} \quad \psi_{1R} \rightarrow e^{-i\alpha/2}\psi_{1R}, \\
&\psi_2 \rightarrow \psi_2.
\end{aligned}
\tag{4.45}
$$

In the spontaneously broken phase, where

$$\phi^\pm = \frac{v}{\sqrt{2}} \sin\left(\frac{\theta}{v}\right),$$

$$\phi^0 = v \cos\left(\frac{\theta}{v}\right), \tag{4.46}$$

the fermions acquire masses

$$m_1^2 = \frac{h_1^2 v^2}{2} \sin^2\left(\frac{\theta}{v}\right),$$

$$m_2^2 = h_2^2 v^2 \cos^2\left(\frac{\theta}{v}\right). \tag{4.47}$$

The one-loop effective potential generated by fermion loops is then

$$\begin{aligned}
V_{1F} &= -\frac{1}{16\pi^2} m_1^4 \ln\left(\frac{m_1^2}{v^2}\right) - \frac{1}{16\pi^2} m_2^4 \ln\left(\frac{m_2^2}{v^2}\right) \\
&= -\frac{1}{64\pi^2} v^4 h_1^4 \sin^4\left(\frac{\theta}{v}\right) \ln\left[\frac{h_1^2}{2} \sin^2\left(\frac{\theta}{v}\right)\right] \\
&\quad -\frac{1}{16\pi^2} v^4 h_2^4 \cos^4\left(\frac{\theta}{v}\right) \ln\left[h_2^2 \cos^2\left(\frac{\theta}{v}\right)\right].
\end{aligned} \tag{4.48}$$

Note that the potential generated by the charged fermions ψ_1 is of the same form as the one-loop gauge potential (4.20), and we can write the full effective potential, including gauge boson loop contributions, in the form

$$\begin{aligned}
V(\theta) &= \frac{v^4}{64\pi^2} \left(3g^4 - h_1^4\right) \sin^4\left(\frac{\theta}{v}\right) \ln\left[\sin^2\left(\frac{\theta}{v}\right)\right] \\
&\quad + \frac{v^4}{64\pi^2} \left[3g^4 \ln\left(g^2\right) - h_1^4 \ln\left(\frac{h_1^2}{2}\right)\right] \sin^4\left(\frac{\theta}{v}\right) \\
&\quad - \frac{v^4}{16\pi^2} h_2^4 \cos^4\left(\frac{\theta}{v}\right) \ln\left[h_2^2 \cos^2\left(\frac{\theta}{v}\right)\right] \\
&\quad - \frac{v^4}{64\pi^2} \left[3g^4 \ln\left(g^2\right) - h_1^4 \ln\left(\frac{h_1^2}{2}\right)\right].
\end{aligned} \tag{4.49}$$

The only effect of the coupling of ϕ to the charged fermion ψ_1 is to add a correction to the vacuum energy density $\Lambda^4 \equiv V(0)$:

$$\Lambda^4 = -\frac{v^4}{64\pi^2} \left[3g^4 \ln\left(g^2\right) - h_1^4 \ln\left(\frac{h_1^2}{2}\right) + 4h_2^4 \ln\left(h_2^2\right)\right]. \tag{4.50}$$

For $h_1 \gtrsim g$, the potential changes sign, and the minimum is at $\theta = 0$. However, we exclude this parameter region from consideration because the mass of the gauge boson A_μ (4.18)

vanishes at the origin, and the resulting theory contains unacceptable long-range forces. If we take $h_1 \ll g$, the correction to the potential due to the fermion ψ_1 is negligible, and we can write the potential as

$$
V(\theta) = \frac{3v^4}{64\pi^2} \left\{ g^4 \sin^4\left(\frac{\theta}{v}\right) \ln\left[g^2 \sin^2\left(\frac{\theta}{v}\right)\right] - g^4 \ln\left(g^2\right) \right\}
$$
$$
- \frac{v^4}{16\pi^2} h^4 \cos^4\left(\frac{\theta}{v}\right) \ln\left[h^2 \cos^2\left(\frac{\theta}{v}\right)\right], \tag{4.51}
$$

where $h \equiv h_2$. For $(\theta/v) \ll 1$, the coupling to the neutral fermion ψ_2 introduces quadratic terms into the potential:

$$
V(\theta) \simeq \frac{v^4}{64\pi^2} \left[3g^4 \ln\left(g^2\right) \left(\frac{\theta}{v}\right)^4 + 8h^4 \ln\left(h^2\right) \left(\frac{\theta}{v}\right)^2 \right]
$$
$$
+ \frac{v^4}{64\pi^2} \left\{ 3g^4 \left(\frac{\theta}{v}\right)^4 \ln\left[\left(\frac{\theta}{v}\right)^2\right] \right\}
$$
$$
- \frac{v^4}{64\pi^2} \left[3g^4 \ln\left(g^2\right) + 4h^4 \ln\left(h^2\right) \right] \quad (\theta \ll v). \tag{4.52}
$$

However, for $h \ll g$, there is still a range of symmetry breaking scales for which the potential is dominated by terms of order $(\theta/v)^4$ at $\theta = \theta_{60}$:

$$
3g^4 \ln\left(g^2\right) \left(\frac{\theta_{60}}{v}\right)^4 > 8h^4 \ln\left(h^2\right) \left(\frac{\theta_{60}}{v}\right)^2. \tag{4.53}
$$

Using the lowest order result for θ_{60},

$$
\left(\frac{\theta_{60}}{v}\right) = \frac{1}{2}\sqrt{\frac{\pi}{15}} \left(\frac{v}{m_{Pl}}\right), \tag{4.54}
$$

we obtain a lower limit on the symmetry breaking scale

$$
\left(\frac{v}{m_{Pl}}\right) > \left(\frac{h}{g}\right)^2 \sqrt{\frac{160 \ln\left(h^2\right)}{\pi \ln\left(g^2\right)}}, \tag{4.55}
$$

which is just the condition (3.47). For $g \simeq 10^{-3}$, taking $h \simeq 10^{-6}$ gives a lower limit $v \gtrsim 10^{-5} m_{Pl} \simeq 10^{14}\text{GeV}$, low enough for symmetry breaking at the grand unified scale, $m_{GUT} \simeq 10^{16}\text{GeV}$.

V. CONCLUSIONS

For scalar field potentials $V(\phi)$ which possess an unstable equilibrium at the origin and a minimum characterized by a symmetry breaking scale v, we have shown that for $v \ll m_{Pl}$, the entire period of inflation is characterized by $\phi \ll v$, and the potential can be expressed in the form (3.5). For the case $m = 2$, the observable quantities produced during inflation are then given by equations (3.41). The COBE limit on the scalar spectral index, $n_s \geq 0.6$, places a lower limit on the effective symmetry breaking scale $(\mu/m_{Pl}) > 0.4$. In addition, the density fluctuation amplitude δ depends exponentially on (μ/m_{Pl}), and fine-tuning of the parameter Λ is required to suppress production of density fluctuations to the level of the COBE value $\delta \simeq 10^{-5}$. For potentials with a vanishing second derivative at the origin, $m > 2$, the corresponding result is given by equations (3.42). In this case, he scalar spectral index n_s is *independent* of any characteristic of the potential except the order of the lowest non-vanishing derivative, and is nearly scale-invariant for all $m > 2$, with $0.93 < n_s < 0.97$. For the case $m = 4$, the density fluctuation amplitude is independent of (μ/m_{Pl}), and there is no intrinsic lower bound on μ from inflationary constraints. For $m > 4$, the density fluctuation amplitude δ *decreases* with decreasing (μ/m_{Pl}), and no fine-tuning of Λ is required. Potentials which contain quadratic terms can still be dominated by terms of order $m > 2$ if the condition (3.45) on the second order slow-roll parameter is met. This allows placement of a lower limit on (v/m_{Pl}).

These results are illustrated by a model in which the inflationary potential is generated by gauge boson loop effects in a Lagrangian with an explicitly broken SO(3) symmetry. In this model, the potential is dominated by terms of order $m = 4$, but its fourth derivative, describing the scalar particle self-coupling, diverges logarithmically at the origin. However, the general analysis is valid up to logarithmic corrections to the limit on the coupling constant obtained from the COBE observation $\delta \simeq 10^{-5}$:

$$g^4 \ln\left(g^2\right) = g_0^4 \ln\left(g_0^2\right) \left(1 + \frac{\ln\left[(\pi/60)(v/m_{Pl})^2\right]}{\ln\left(g_0^2\right)}\right)^{-1}$$

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$$g_0^4 \ln \left(g_0^2 \right) = -\frac{8}{3} \left(\frac{\pi^2}{15} \right)^3 \delta^2, \tag{5.1}$$

where $g \simeq 10^{-3}$ for a wide range of scales v. The spectral index n_s is given exactly by the value derived in the general analysis, $n_s = 0.95$, independent of the symmetry breaking scale. Couplings to fermionic sectors are discussed. In a model with fermions, in which the Yukawa couplings also break the SO(3) symmetry, quadratic terms are introduced into the potential, and a lower bound on (v/m_{Pl}) is obtained:

$$\left(\frac{v}{m_{Pl}} \right) > \left(\frac{h}{g} \right)^2 \sqrt{\frac{160 \ln \left(h^2 \right)}{\pi \ln \left(g^2 \right)}}, \tag{5.2}$$

where h is the Yukawa coupling to a neutral fermion. For weakly coupled fermions, $h \simeq 10^{-6}$, inflation is consistent with a symmetry breaking scale $v \simeq m_{GUT} \simeq 10^{16}$GeV. Models with SO(3) symmetric fermion sectors possess no such lower limit.

These results in many respects do not bode well for efforts at "reconstruction" – the determination of specific details – of the inflationary potential from accurate measurement of the scalar spectral index and tensor fluctuation amplitude [9,22–31]. For a large class of viable models, the tensor fluctuation amplitude is very small, and other cosmological constraints are largely insensitive to the specific form of the scalar field potential, leaving little opportunity to distinguish one model from another using cosmological observations alone.

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REFERENCES

[1] S.W. Hawking, Phys. Lett. B **115**, 295 (1982).

[2] A. Starobinsky, Phys. Lett. B **117**, 175 (1982).

[3] A. Guth and S.Y. Pi, Phys. Rev. Lett. **49**, 1110 (1982).

[4] J.M. Bardeen, P.J. Steinhardt and M.S. Turner, Phys. Rev. D **28**, 679 (1983).

[5] W.H. Kinney and K.T. Mahanthappa, Report No. hep-ph/9511460 (unpublished).

[6] A. Guth, Phys. Rev. D **23**, 347 (1981).

[7] A.D. Linde, Phys. Lett. B **108**, 389 (1982).

[8] A. Albrect and P.J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).

[9] E.J. Copeland, E.W. Kolb, A.R. Liddle and J.E. Lidsey, Phys. Rev. D **48**, 2529 (1993). This paper contains an up-to-date review of slow-roll inflation and extensive relevant references.

[10] E.D. Stewart and D.H. Lyth, Phys. Lett. B **302**, 171 (1993).

[11] G.F. Smoot et. al., Astrophys. J. **396**, L1 (1992).

[12] E.L. Wright et. al., Astrophys. J. **396**, L13 (1992).

[13] C.L. Bennett, et. al., Astrophys. J. **436**, 423 (1994).

[14] C.L. Bennett et. al., "COBE-DMR Two-Year Large Scale Anisotropy Results," in Report No. astro-ph/9502032 (unpublished).

[15] K. Freese, J.A. Freeman and A.V. Olinto, Phys. Rev. Lett. **65**, 3233 (1990).

[16] F.C. Adams, J.R. Bond, K. Freese, J.A. Frieman and A.V. Olinto, Phys. Rev. D **47**, 426 (1993).

[17] W.H. Kinney and K.T. Mahanthappa, Phys. Rev. D **52**, 5529 (1995).

[18] L. Knox and A. Olinto, Phys. Rev. D **48**, 946 (1993).

[19] P. Parsons and J.D. Barrow, Phys. Rev. D **51**, 6757 (1995), have derived a similar expression for the case of an exponential potential. Here we have derived the result independent of any particular physical context.

[20] A.D. Linde, *Particle Physics and Inflationary Cosmology* (Harwood Academic Publishers GmbH, Chur, Switzerland, 1990), Chap. 8, derives this result for the case m = 4.

[21] S. Weinberg, Phys. Rev. D **7**, 2887 (1973).

[22] H.M. Hodges and G.R. Blumenthal, Phys. Rev. D **42**, 3329 (1990).

[23] D.S. Salopek, Phys. Rev. Lett. **69**, 3602 (1992).

[24] F. Lucchin, S. Matarrese, and S. Mollerach, Astrophys. J. Lett. **401**, 49 (1992).

[25] A.R. Liddle and D.H. Lyth, Phys. Lett. B **291**, 391 (1992).

[26] R.L. Davis, H.M. Hodges, G.F. Smoot, P.J. Steinhardt, and M.S. Turner, Phys. Rev. Lett. **69**, 1856 (1992).

[27] E.J. Copeland, E.W. Kolb, A.R. Liddle, J.E. Lidsey, Phys. Rev. Lett. **71**, 219 (1993).

[28] M.S. Turner, Phys. Rev. D **48**, 5539 (1993).

[29] E.J. Copeland, E.W. Kolb, A.R. Liddle, J.E. Lidsey, Phys. Rev. D **49**, 1840 (1994).

[30] A.R. Liddle and M.S. Turner, Phys. Rev. D **50**, 758 (1994).

[31] F.C. Adams and K. Freese, Phys. Rev. D **51**, 6722 (1995).